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A DYNAMIC PROGRAMING APPROACH TO ARMY FORCE PLANNING

OCTOBER 1975

PREPARED BY

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A joint Concepts Analysis Agency - United States Military Academy study made to describe and determine the feasibility of use of a mathematical technique called Dynamic Programing for Army force planning is reported. The paper is primarily tutorial and is aimed at the practitioner and user of analytical studies. Dynamic Programing is a mathematical method or algorithm that effectively solves sequential decision problems. The principle of optimality is employed. The algorithm shows the decision maker the sequence of decisions to make to maximize the return function. The Dynamic Programing method is suited for force planning

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and offers significant advantages over trial and error or linear programing solutions.

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FOREWORD

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Colonel Richard M. Meyer of the Concepts Analysis Agency (CAA) originally suggested a joint CAA - United States Military Academy (USMA) research project for the summer of 1975. His vigorous and enthusiastic support of the concept and his ability to overcome the administrative obstacles were keys to the success of this work.

Major General Hal E. Hallgren, the Commanding General of CAA deserves thanks for his personal interest in the project and for ultimate approval of the summer program. It was a new experience for both sides and without command attention, the project would surely have floundered.

Work at CAA was with the Europe Forces Group of the Force Concepts and Design Directorate. Each analyst in the Group was more than generous in giving advice, counsel, and assistance. However, at the risk of some slight the authors would particularly like to mention Colonel Robert E. Robinson and Captain Patrick M. Keating. Colonel Robinson as Chief, Europe Forces Group, is a rare blend of experienced combat troop leader and qualified analyst. He has a perceptive, imaginative, and sound approach to analytical problems and the communication skills to explain his approach to even the most naive listener. Captain Keating represents the "new wave" of analysts. He is a superb and dedicated technician. His delight in his work is coupled with a surprisingly mature approach to analytical problems. While we certainly hope for a _quid pro quo_ relationship with CAA, we must admit our learning debt to these two individuals is great.

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A DYNAMIC PROGRAMING APPROACH TO ARMY FORCE PLANNING

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A DYNAMIC PROGRAMMING APPROACH TO ARMY FORCE PLANNING

SUMMARY

1. Purpose

The purpose of this paper is to describe a joint Concepts Analysis Agency (CAA) - United States Military Academy (USMA) study to determine the feasibility of the use of a mathematical technique known as Dynamic Programing for Army force planning.

2. Scope

The thrust of this paper is primarily tutorial and is aimed at the practitioner as well as the user of analytical studies. A description of the Dynamic Programing methodology is presented. While not unique, this discription is sufficiently distinctive to afford even an experienced analyst some valuable insights.

3. Background

Since its inception in 1973 CAA has been concerned with the conceptual design of the Army in the field (CONAF). In previous pioneering studies CAA defined the "Base Case" force. The Base Case is used in CONAF studies as a point of departure for examining conceptual alternative forces. For CONAF IV, fiscal year (FY) 87 was selected and the end FY 74 force was projected through FY 87 to establish the Base Case. The projection reflects the application of identified plans and programs to the current force and provides a force continuum for the period. The process is essentially one of force modernization within the constraints of resource projections, consistent with major procurement and force structure plans.

4. Methodology

Dynamic Programing is a mathematical method or algorithm that effectively solves sequential decision problems. The principle of optimality is employed. The algorithm shows the decision maker the sequence of decisions to maximize the return function. The value of the maximum return function is unique although there may be any number of policies (decisions in the sequence) that yield this same value.

5. Limitations

a. Multiple constraints may lead to technical problems in the solution procedure that have not yet been solved through this research.

b. A completely satisfactory theoretical approach to cope with multiattributed criteria is nonexistent.

6. Assumptions

a. The problem is one in which sequential decisions can be made.

b. The courses of action are identifiable and have associated outcomes each of which has a finite payoff.

7. Evaluation Criteria. - Maximize the return function.

8. Observations

a. The following general observations were made during the conduct of the joint USMA-CAA study of a Dynamic Programing approach to Army force planning.

(1) The Dynamic Programing method, in general, is suited for force planning and offers significant advantages over trial and error or linear programing solutions.

(2) There is a class of possible problems involving multiple objectives and/or multiple constraints that are not susceptible to Dynamic Programing solutions at the present state of USMA-CAA development.

(3) Future extensions suggested for CAA considerations are: Expansion of criteria and decision variables, improvement in cost and effectiveness factors, and modification of computer programs.

(4) The CAA cost and effectiveness data base is not entirely satisfactory for either linear programing or Dynamic Programing solutions. A systematic analysis of the CEM runs is needed to precisely evaluate cost and effectiveness factors. With these improvements CAA could well use Dynamic Programing routinely in future CONAF studies.

b. Specific observations were made in the application of Dynamic Programing approach to the "Solution of a Sample Problem," and in the determination of "A European Oriented Optimal Force."

(1) "Solution of a Sample Problem".

a. Dynamic Programing always tells the decision maker more than just the answer to one specific question and provides a veritable storehouse of information for post hoc analysis or answers to "what if" questions.

b. Almost unlimited variations of single constraints could be considered when Dynamic Programing is used to determine, for example, optimal force mix.

c. If the analyst is astute in his choice of decision variables, the answers will be in terms completely familiar to the decision maker. Translation problems are not encountered and all constraints are rigorously met.

(2) "A European Oriented Optimal Force".

a. The two constraints of the European force problem--a fixed dollar amount of resources and a range of deviations of the decision variables--were well suited to the Dynamic Programing solution.

b. In this exercise tank kills are maximized by increasing the number of tank battalions in the authorized force.

c. The alternative optimal force is relatively insensitive to the type of curve fit used. Although an exponential curve has mathematically satisfying properties indications are that an analyst should not be reluctant to turn to the expedient linear fit.

d. Only for the hyperbolic curve fit case, where the artillery battalions dip to 148, is any constraint "tight." Part of this may be attributed to "round-off" of costs in the Dynamic Programing solution. There is a mild presumption that loosening the constraint, perhaps to ± 30 percent, or even ± 50 percent, would not result in a radically different force.

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A DYNAMIC PROGRAMING APPROACH TO ARMY FORCE PLANNING

CHAPTER I
INTRODUCTION

1. <u>The Base Case Force</u>

The Base Case provides a reasonably well understood framework for evaluating the performance characteristics and resource implications of any conceptual force. In addition, it provides to the development community a long range force, with a general scenario, that can be used as a medium for evaluating operational and materiel system concepts. The Base Case force is evaluated by the CONAF Evaluation Model (CEM) as shown in Figure 1. The CEM is a computer simulation utilizing a European scenario. Figure 1 is meant to merely suggest the hundreds of output data that are generated by the CEM.

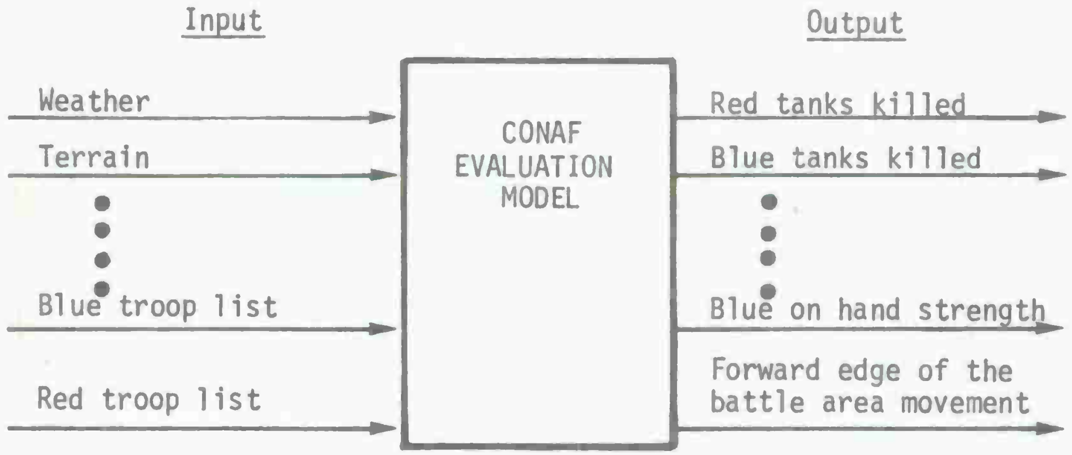

FIGURE 1, CONAF Evaluation Model

2. <u>Alternative Criteria</u>

a. Implicit in all CONAF studies has been the design of a force that is "better" than the Base Case force. In 1975, for example, CAA designed alternative forces that:

(1) Killed more Red tanks but cost no more than the Base Case.

(2) Killed more Red personnel but cost no more than the Base Case.

(3) Equaled the Base Case in Red tank and personnel kills but minimized the Blue personnel force on the battlefield.

b. Other objectives that a decision maker might conclude define a "better" force are:

(1) Limit forward edge of the battle area (FEBA) movement to some number of kilometers.

(2) Minimize the size of the Blue tank force subject to some effectiveness and cost constraints.

(3) Minimize logistics support or sea lift required.

(4) Maximize the Red to Blue kill ratio subject to a cost constraint.

(5) Provide a minimum cost Blue force that will last some set number of days; for example, 30 days.

(6) Some combination of the above.

c. Overlaying all these criteria are the questions: Against whom? When? For how long? Where? Certainly a force that would maximize tank kills in the Sinai desert for a 30-day war may be different than the force that would maximize tank kills in Germany for a 100-day war.

3. <u>Previous Methodology</u>

a. The role of the decision maker is to determine the criteria for the "better" force. The role of the analyst is to design the force. The early CONAF efforts were trial and error exercises to meet the specified criteria. However, the CEM is an incredibly complicated model, and some of the trials resulted in alternative forces that were actually inferior to the base case. Even when the trial was "success ful" in that it produced an alternative force that, for example, killed more Red tanks than the Base Case, there was no guarantee that the alternative was even close to being an "optimal" force. In other words, there might have existed another force that not only cost no more than the Base Case but also was far superior in tank kills to the designed alternative.

b. In 1975, CAA made a significant advance to remedy these faults by using a method known as linear programing to arrive at alternative forces. Linear programing has these advantages:

(1) Simple problem formulation.

(2) Extremely efficient computer solutions.

(3) Ability to handle numerous constraints.

c. Conversely, linear programing has numerous shortcomings that CAA recognized.

(1) Linear cost assumption.- There is a strong reluctance to believe that addition of one tank battalion to the Base Case at a hypothetical cost of $45 million implies that the addition of ten tank battalions would increase the cost ten times or $450 million.

(2) Linear effectiveness.- A second assumption is that if one tank battalion added to the Base Case yields an increase of 30 Red tanks killed, then the addition of ten tank battalions would result in an increase of 300 Red tank kills.

(3) Translation problems.- The results of the linear programing were sets of percentages. A typical solution might be:

Item change	Percent change
tanks	+20
armored personnel carrier	-18
helicopters	+10
mortars	-05
artillery tubes	00
antitank weapons	+20

The first difficulty is the integer problem as a 20 percent increase in tanks may imply an increase of 105.63 tanks, an obvious physical impossibility. In such cases, round off neither guarantees optimality nor satisfaction of the cost constraint. In addition, the Base Case is described as a certain number of tank battalions, artillery battalions, helicopter companies, etc. As a result, CAA had some difficulty in translating from percentages of weapons systems to troop units, and the translation, at times, actually degraded desired performance and violated constraints.

d. The joint CAA-USMA study was performed in an attempt to overcome these shortcomings by use of Dynamic Programing.

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CHAPTER II
THE DYNAMIC PROGRAMING METHODOLOGY

1. What is Dynamic Programing?

Dynamic Programing is a mathematical method or algorithm that effectively solves sequential decision problems. The term "sequential decisions," as used here means a series of decisions where the later decisions are dependent on the earlier decisions. Dynamic Programing follows a middle course between opportunism and conservation. An opportunist might use all available resources at early decisions only to miss better opportunities at later decisions. Conversely, the conservationist may maintain vast resources and not have sufficient decisions remaining in which to commit these resources. Dynamic Programing affords early recognition of opportunities but husbands resources for later opportunities. In this document these abstracts are illustrated with a sample force planning problem.

2. The Theoretical Basis

a. An elementary problem to illustrate the theoretical basis is given. Assume the availability of a $10 million budget to achieve an optimal force of infantry and armor brigades and cavalry regiments that will maximize kill of enemy tanks. Costs of the units and the returns for this sample problem are as shown in Tables 1 through 3.

TABLE 1. Infantry Brigades
(Cost and returns for sample problem)

Brigades (number)	Dollar cost (millions)	Return (enemy tanks)
0	0.0	0
1	1.5	20
2	3.0	40
3	5.0	70
4	6.5	90
5	8.0	110

TABLE 2. Armor Brigades
(Cost and returns for sample problem)

Brigades (number)	Dollar cost (millions)	Return (enemy tanks)
0	0.0	0
1	3.0	40
2	6.0	75
3	10.0	130

TABLE 3, Cavalry Regiments
(Cost and returns for sample problem)

Brigades (number)	Dollar cost (millions)	Return (enemy tanks)
0	0.0	0
1	2.0	35
2	4.0	70

b. Picture a large number of points in space that represent the possible combinations of infantry, armor, and cavalry units and uncommitted dollars obtainable with a $10 million budget. For example, points reflecting the following acquisitions and remaining resources might be:

(1) Two infantry brigades, zero armor brigades, one cavalry regiment and $5 million in uncommitted resources.

(2) One infantry brigade, one armor brigade, one cavalry regiment, and $3.5 million in uncommitted resources.

c. A logical start in this sample problem is the point where there are zero infantry brigades, zero armor brigades, zero cavalry regiments, and $10 million in resources. First a decision of how many infantry brigades to buy will be made. The alternatives are shown in Figure 2. The mathematical term for this move from the starting point to each of the six choices of Figure 2 is a transformation. The six choices are identified as T_1, T_2, T_3, T_4, T_5, and T_6.

Several generalizations can be made about transformations.

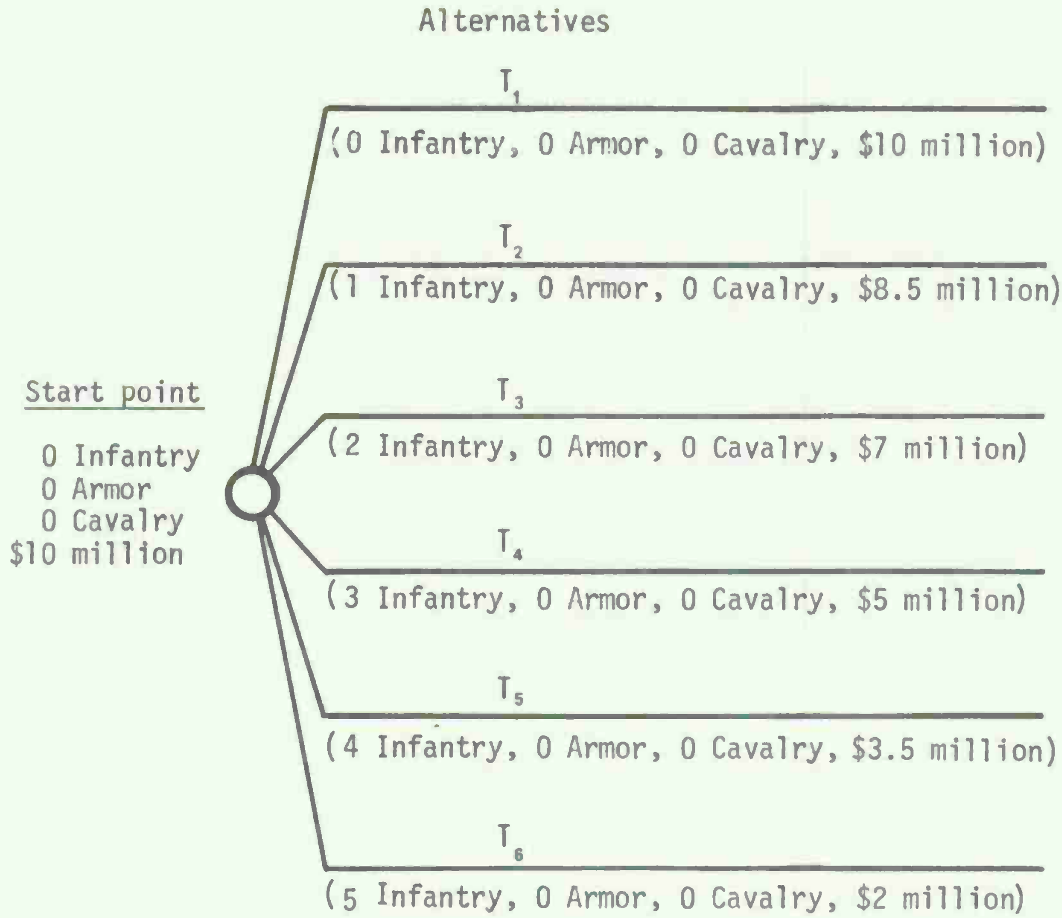
- A transformation must be bought, i.e., movement from one point to a second has an associated cost (the cost for one alternative is zero).

- Each transformation moves to a unique succeeding point

- Each transformation (except one) yields an increase in the return function, in this case, tanks killed.

- For the force planning problem (although not in the general Dynamic Programing formulation) each transformation can be used only once.

Note that the opportunist would buy five infantry brigades while the conservationist would save resources for the purchase of the armor brigades and the cavalry regiments. As will be shown both or either of these alternatives may reflect nonoptimal decisions.



Alternatives: T_1, T_2, T_3, T_4, T_5, and T_6

FIGURE 2, Infantry Alternatives

d. Assume that a decision has been made to buy a specified number of infantry brigades. There exists a completely different set of transformations that moves the planner from each of the six solution points of the first decision (the number of infantry brigades to buy) to this second point. Each transformation has a cost, and each has an associated return of enemy tanks killed. Example: a move from the point where assets equaled two infantry brigades, zero armor brigades, zero cavalry regiments, and $7 million of uncommitted funds (with a return of 40 tanks killed) to a point where assets consist of two infantry brigades, one armor brigade, zero cavalry regiments, and $4 million of uncommitted resources (with a total return of 80 tanks killed). The problem is completed by deciding on the number of cavalry regiments. The complete process is depicted in Figure 3.

e. The Dynamic Programing algorithm is a roadway through the labyrinth of Figure 3. This algorithm shows the decision maker which of the many numerous paths to take that will maximize the return function (enemy tanks killed). There may be several routes to the same destination.

f. Two assertions occur frequently in Dynamic Programing literature and are included here as a matter of completeness. The first is the principle of optimality. Colloquially this means that irrespective of how the decision maker got to some point in a solution, subsequent decisions must be optimal with respect to that solution point. The second assertion is no more than a restatement of the preceeding paragraph. This assertion is that the value of the maximum return function is unique; however, there may be any number of policies, i.e., decisions in the sequence, that yield this same value.

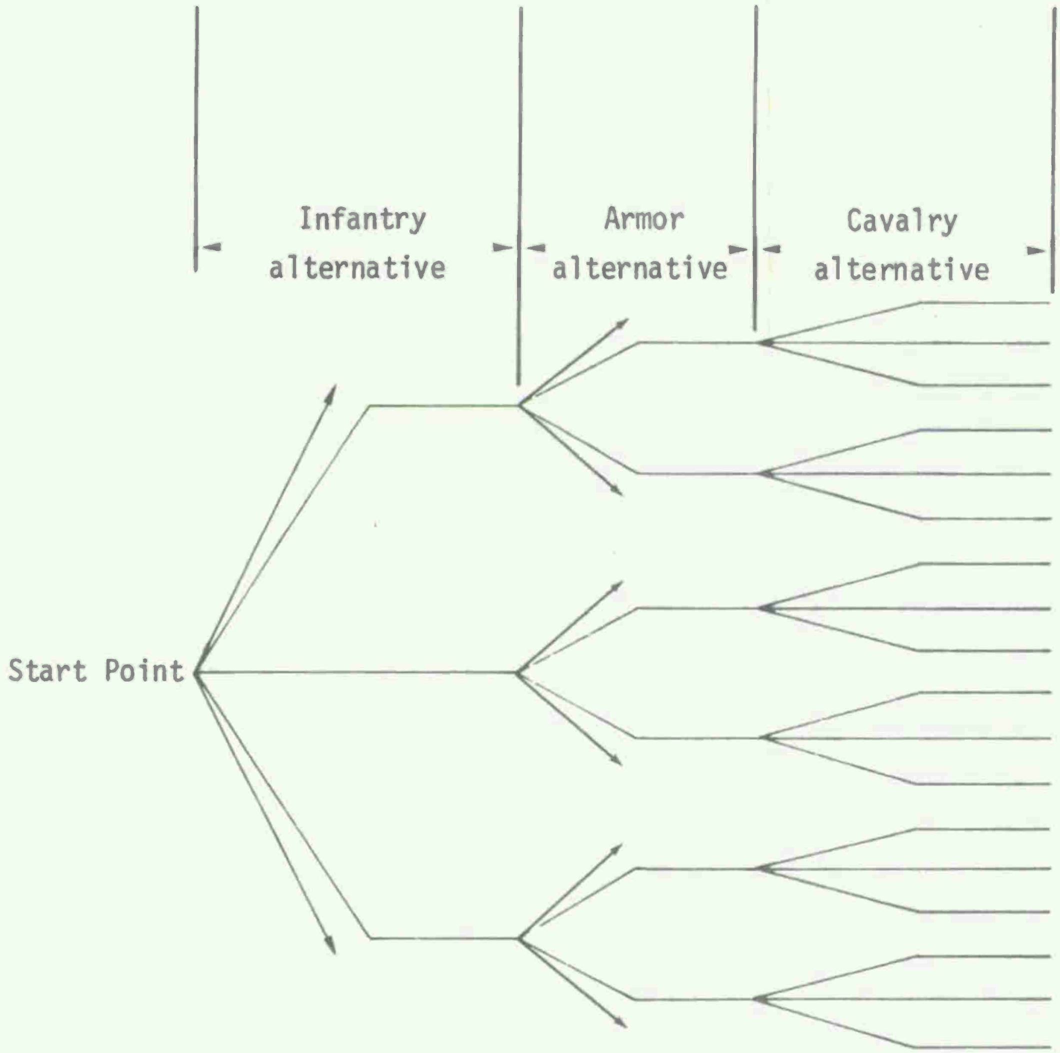


FIGURE 3, Infantry, Armor, and Cavalry Alternatives

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CHAPTER III
THE SOLUTION OF A SAMPLE PROBLEM

1. General Discussion

The techniques described are illustrated by presentation of a complete solution of the elementary problem posed to illustrate the theoretical basis. The Dynamic Programing solution informs the decision maker what mix of infantry and armor brigades and cavalry regiments will maximize enemy tank kills, subject to the constraint that the budget for this purpose cannot exceed $10 million.

2. Notation

a. A minimal amount of notation is defined here to hasten the solution process.

b. Assume three decision variables, the number of infantry and armor brigades and cavalry regiments. Let,

X_1 = number of infantry brigades

X_2 = number of armor brigades

X_3 = number of cavalry regiments

For memory purposes, let \$ represent the number of dollars available to the decision maker. Recall a point in the solution is defined by giving a value to X_1, X_2, X_3, and \$. Let $\underline{s}$ collectively represent the four values of this point and $f(\underline{s})$ represent the value of the return function at the point $\underline{s}$. Use an asterisk to represent optimal values. For example, X_2^* represents the optimal number of armor brigades.

3. The "First" Decision

a. Part of the wizardry of Dynamic Programing is the conceptual jump from looking at the decisions in sequential order 1, 2, 3 to consideration of the decisions in reverse order 3, 2, 1. The solution begins by tabulating various possibilities that could occur at the last stage or decision. Suppose, for example, the decision maker arrived at the last stage or decision with zero dollars. How many infantry brigades would the analyst recommend buying and what would be the value of the return function? The obvious answers are zero and zero.

b. Assume now the decision maker had $6 million. Zero, one, two, or three infantry brigades could be bought but not four or five as shown in Tables 1 through 3. The rational decision would be to buy three infantry brigades and receive a return of 70 tanks killed. The analysts would repeat the process for $0 to $10 million.

c. For "hand" solution, the analyst frequently finds a tabular presentation, such as Table 4, a valuable assist in organizing a solution. Example of table usage: With $7.5 million in uncommitted resources, the decision maker can choose from the following.

(1)

Number of brigades	Tanks killed
0	0
1	20
2	40
3	70
4	90

(2) The decision maker may not buy five brigades as this number costs more than $7.5 million. The optimal policy is to buy four infantry brigades.

TABLE 4, Optimal Policies for Purchase of Infantry Brigades

X_1 / $ Dollars	Infantry brigades 0	1	2	3	4	5	$f_1^*(\underline{s})$	X_1^*
0.0	0	a/	a/	a/	a/	a/	0	0
0.5	0	a/	a/	a/	a/	a/	0	0
1.0	0	a/	a/	a/	a/	a/	0	0
1.5	0	20	a/	a/	a/	a/	20	1
2.0	0	20	a/	a/	a/	a/	20	1
2.5	0	20	a/	a/	a/	a/	20	1
3.0	0	20	40	a/	a/	a/	40	2
3.5	0	20	40	a/	a/	a/	40	2
4.0	0	20	40	a/	a/	a/	40	2
4.5	0	20	40	a/	a/	a/	40	2
5.0	0	20	40	70	a/	a/	70	3
5.5	0	20	40	70	a/	a/	70	3
6.0	0	20	40	70	a/	a/	70	3
6.5	0	20	40	70	90	a/	90	4
7.0	0	20	40	70	90	a/	90	4
7.5	0	20	40	70	90	a/	90	4
8.0	0	20	40	70	90	110	110	5
8.5	0	20	40	70	90	110	110	5
9.0	0	20	40	70	90	110	110	5
9.5	0	20	40	70	90	110	110	5
10.0	0	20	40	70	90	110	110	5

Uncommitted resources

a/ Insufficient resources for purchases.

4. <u>The Second Stage or Decision</u>

a. Now consider the number of armor brigades to purchase. The techniques can best be illustrated now by assuming that the decision maker has $9.5 million in uncommitted resources at this stage. The decision maker has these options:

(1) Buy zero armor brigades and go to stage 3 with $9.5 million. Examination of Figure 4, an excerpt from Table 4, shows that the optional policy would be to buy five infantry brigades and receive a return of 110 tanks killed.

Infantry brigades

$ Dollars \ X_1	0	1	2	3	4	5	$f^*(\underline{s})$	X_1^*
9.0							110	5
9.5	0	20	40	70	90	110	110	5
10.0							110	5

FIGURE 4, Optimal Policy With $9.5 Million

(2) Buy one armor brigade for $3 million. This yields a return of 40 tanks killed and an uncommitted fund of $6.5 million for stage 3. Figure 5, another excerpt from Table 4, shows the new optimal decision is to buy four infantry brigades with the $6.5 million. The return for this policy is 130 tanks killed (40 from the armor brigade and 90 from the infantry brigades).

Infantry brigades

$ Dollars \ X_1	0	1	2	3	4	5	$f^*(\underline{s})$	X_1^*
6.0							70	3
6.5	0	20	40	70	90	<u>a</u>/	90	4
7.0							90	4

<u>a</u>/ Insufficient resources for purchase

FIGURE 5, Optimal Policy With $6.5 Million

(3) Buy two armor brigades for $6 million and receive a return of 75 tanks killed. With the remaining $3.5 million the decision maker would optimally purchase two infantry brigades for a return of 40 tanks killed (see Figure 6) or a total of 115 tanks killed.

Infantry brigades

$ \ X_1	0	1	2	3	4	5	f(s)	X_1*
3.0				a/	a/	a/	40	2
3.5	0	20	40	a/	a/	a/	40	2
4.0				a/	a/	a/	40	2

a/ Insufficient resources for purchase

FIGURE 6, Optimal Policy With $3.5 Million

b. Note that the assumption of $9.5 million of uncommitted funds precludes purchase of more than two armor brigades. Therefore, the solution points are limited to the three stated above. Naturally, the decision maker would be expected to elect the second alternative to achieve 130 tanks killed. The analyst conducts a similar process for all values from 0 to $10 million to complete Table 5.

5. The "Last" Decision

a. The consideration of the decision that would be first in sequential order but last in the Dynamic Programing technique proceeds in exactly a parallel manner. The authors assume that the discussion to this point has sufficient clarity that the reader can mentally construct Table 6.

b. The optimal solution for the entire problem is read from the bottom row of Table 6, and proceeding back through Tables 4 and 5. The decision maker should spend the entire $10 million to buy two calvary regiments, one armor brigade, and two infantry brigades, and receive a return of 150 tanks killed.

6. Findings

a. Mention of several findings appears appropriate at this point. Some interest is found in tracing the decision behavior of the hypothetical opportunist and conservationist. The opportunist would spend $4 million to buy two cavalry regiments at state, or decision, 1 and the remaining $6 million at stage 2 for two armor brigades and realize a return of 145 tanks killed--less than the 150 tanks killed in the optimal solution. The conservationist would not spend money until

TABLE 5, Optimal Policies at Stage 2

X_2 / $ Dollars	Armor Brigades 0	1	2	3	$f_2^*(\underline{s})$	X_2^*
0.0	0+0	a/	a/	a/	0	0
0.5	0+0	a/	a/	a/	0	0
1.0	0+0	a/	a/	a/	0	0
1.5	0+20	a/	a/	a/	20	0
2.0	0+20	a/	a/	a/	20	0
2.5	0+20	a/	a/	a/	20	0
3.0	0+40	40+0	a/	a/	40	0,1
3.5	0+40	40+0	a/	a/	40	0,1
4.0	0+40	40+0	a/	a/	40	0,1
4.5	0+40	40+20	a/	a/	60	1
5.0	0+70	40+20	a/	a/	70	0
5.5	0+70	40+20	a/	a/	70	0
6.0	0+70	40+40	75+0	a/	80	1
6.5	0+90	40+40	75+0	a/	90	0
7.0	0+90	40+40	75+0	a/	90	0
7.5	0+90	40+40	75+20	a/	95	2
8.0	0+110	40+70	75+20	a/	110	0,1
8.5	0+110	40+70	75+20	a/	110	0,1
9.0	0+110	40+70	75+40	a/	115	2
9.5	0+110	40+90	75+40	a/	130	1
10.0	0+110	40+90	75+40	130+0	130	1,3

a/ Insufficient resources for purchase.

stage, or decision, 3. The conservationist would then spend $8 million for five infantry brigades, receive a return of 110 tanks killed, and regret that $2 million of funds remained uncommitted.

b. Note that neither the costs nor the returns shown in Tables 1 through 3 are linear. Thus, one drawback of linear programing has been effectively overcome.

c. Dynamic Programing always serves as a means to furnish more information to the decision maker than just the answer to one specific question. Dynamic Programing provides a veritable storehouse of information for post hoc analysis or answers to "what if" questions. As an example: "What if the budget were reduced to $8 million"? Table 6 (and back stepping through Tables 4 and 5) immediately reveals that the optimal solution is one cavalry regiment, one armor brigade, and two infantry brigades, for a return of 115 enemy tanks killed.

d. With a minimal effort Table 6 could be expanded beyond $10 million to permit study of the effects of possible budget increases. The perceptive analyst would also note that if the objective were to kill at least 90 enemy tanks a force could be designed that would be capable of meeting that criteria for either $5.5 or $6 million.

e. Almost unlimited variations of constraints could be considered; variations such as, "suppose doctrine called for at least one of each type brigade"? What then is the optimal mix? Finally, the answers are in terms completely familiar to the decision maker--so many infantry brigades, so many armor brigades, so many cavalry regiments. Translation problems are eliminated and all constraints are rigorously met.

TABLE 6, Optimal Policies at Stage 1

X_3 / $ Dollars	Cavalry Regiments 0	1	2	$f_3^*(\underline{s})$	X_3^*
0.0	0+0	a/	a/	0	0
0.5	0+0	a/	a/	0	0
1.0	0+0	a/	a/	0	0
1.5	0+20	a/	a/	20	0
2.0	0+20	35+0	a/	35	1
2.5	0+20	35+0	a/	35	1
3.0	0+40	35+0	a/	40	0
3.5	0+40	35+20	a/	55	1
4.0	0+40	35+20	70+0	70	2
4.5	0+60	35+20	70+0	70	2
5.0	0+70	35+40	70+0	75	2
5.5	0+70	35+40	70+20	90	2
6.0	0+80	35+40	70+20	90	2
6.5	0+90	35+60	70+20	95	1
7.0	0+90	35+70	70+40	110	2
7.5	0+95	35+70	70+40	110	2
8.0	0+110	35+80	70+40	115	1
8.5	0+110	35+90	70+60	130	2
9.0	0+115	35+90	70+70	140	2
9.5	0+130	35+95	70+70	140	2
10.0	0+130	35+110	70+80	150	2

a/ Insufficient resources for purchases.

NOT USED

CHAPTER IV
A EUROPEAN ORIENTED OPTIMAL FORCE

1. Problem Selection

a. A "real" life problem, the structuring of a force that maximizes Red tank kills for a 96-day war, was chosen. A European scenario that is subject to the following two constraints was used.

(1) The cost is equal to or less than the fiscal year (FY) 87 Base Case.

(2) The combat battalions that represent the decision variables can deviate $\pm$ 20 percent from the Base Case force.

b. The reason for the first constraint is obvious. The second constraint precludes radical excursions from the Base Case that might lead to doctrinal quandries or production difficulties (to meet tank requirements, as an example, if the tank force were doubled.)

c. The problem was well defined and particularly suited to Dynamic Programing. Definition of the problem was facilitated by the fact that the same problem was attacked using linear programing during the 1975 CONAF study.

2. Problem Formulation

a. Problem Statement. - Design a force costing no more than the FY 87 Base Case that will maximize Red tank kills in a 96-day war using an European scenario. The numbers of combat battalions of each type can deviate only $\pm$ 20 percent from the Base Case force.

b. Decision Variables. - Concepts Analysis Agency had formulated the linear programing problem by the use of weapon "slices." There were six slices: Tanks, armored personnel carriers, helicopters, artillery tubes, mortars, and antitank weapons. This approach represented a judicious trade-off between the ease of formulating the linear programing problem and the need to develop a troop list. As previously noted, these decision variables led to subsequent translation problems. Consequently, the following decision variables were chosen for the CAA-USMA study.

(1) Tank battalions

(2) Mechanized battalions

(3) Infantry battalions

(4) Artillery battalions

(5) Armored cavalry squadrons

The problem is not exactly the same as that solved by CAA as combat aviation companies are not included as decision variables. However, from a tutorial perspective, the five decision variables adequately represent the methodology and additional decision variables might confuse the approach.

c. Cost and Effectiveness Factors. - The reader may infer from the sample problem that once an analyst is given the criteria for the problem (maximize tank kills, maximize people killed, etc.) and decides on the decision variables, the solution of the problem is simple--merely arrive at the effectiveness and cost factors and follow a mechanistic path to the solution. However, the determination of the proper cost and effectiveness factors for this problem was neither simple nor mechanistic.

d. Procedures. - The procedures for calculating the cost and effectiveness factors are reasonably complex and are included as Appendix D, "Cost Calculations," and Appendix E, "Effectiveness Factors," to avoid a prolonged break in this narrative.

3. Methods of Solution

a. Method Selected. - The hand solution of a Dynamic Programing problem is always possible. The process, however, reaches a maximum tedium tolerance level at about four decision variables. Therefore, as part of this project a computer program was developed that will accept 30 decision variables and furnish a Dynamic Programing solution. The listing of the program is at Appendix F.

4. The Solution and Comments

a. The Base Case force and the three forces found by the Dynamic Programing computer program are tabulated in Table 7 for easy comparison.

TABLE 7, Alternative Optimal Forces

Force elements	Base Case	Linear fit	Hyperbolic fit	Exponential fit
Tank battalion	85	104	103	104
Mechanized battalion	88	73	75	74
Infantry battalion	89	89	87	87
Artillery battalion	188	149	148	149
Cavalry Squadron	15	17	17	17
Totals	465	432	430	431

b. Efforts have been made to avoid the trap of stating far-reaching conclusions from fragmentary results. However there are several findings, some of which are self-evident, that seem pertinent. The first is: Tank kills are maximized by increasing the number of tank battalions in the authorized force. This increase is accomplished primarily by deletion of artillery battalions and, to a lesser extent, deletion of mechanized and infantry battalions. There is a minor upswing in cavalry squadrons. (This finding is not stated to denigrate the "King of the Battlefield." Opposite results would be expected in a people killing force, i.e., addition of artillery battalions at the expense of tank battalions.)

c. Of greater import is the finding that the alternative optimal force is relatively insensitive to the curve fit. There is no more than a two battalion deviation from case to case. While the exponential curve has mathematically satisfying properties, the limited analysis here indicates that an analyst should not be reluctant to turn to the expedient linear fit. These results also signal that within the considered range ($\pm$ 20 percent of Base Case), the three fitted curves are very nearly indistinguishable.

d. Note that only for the hyperbolic case where the artillery battalions dip to 148 is any constraint "tight." Part of this may be attributed to "round-off" of costs in the Dynamic Programing solution. There is a mild presumption of far greater significance that loosening the constraint (perhaps to $\pm$ 30 percent, or even $\pm$ 50 percent) will not result in a radically different force. Somewhat fewer artillery battalions and more tank battalions might be expected. However, even these deviations may be small around the "stable" solution point.

e. The USMA researchers progressed no further than this point. Promising future extensions for CAA consideration are detailed in Chapter V.

A DYNAMIC PROGRAMING APPROACH TO ARMY FORCE PLANNING

CHAPTER V
EXTENSION AND NEEDED STUDY

1. Extensions

The extensions are conveniently grouped in three categories.

a. Expansion of criteria and decision variables

b. Improvement of cost and effectiveness factors

c. Modification of computer programs.

2. Criteria and Decision Variables

a. A single attributed criteria function, maximize Red tank kills, has been examined. However, there are more complex questions such as: "Which is the best force--one that kills 3000 tanks and 10,000 people or, one that kills 2000 tanks and 20,000 people"? These multiattributed criteria make the analyst's task considerably more difficult, and a completely satisfactory theoretical approach to handle this complexity is nonexistent. In general, the analyst must collapse this multiattributed criteria to some common numeraire. Dollars are suggested for this purpose. Expert opinion (elicited perhaps by the Delphi technique) is one assist. Implied or derived trade-off curves are a second assist. Both the Dynamic Programing solution and the linear programing solution (the "shadow" costs from the dual solution to the linear programing problem) furnish data for such trade-off curves. Such trade-offs have not been considered in this research.

b. Note that the two constraints of the European force problem--a fixed dollar amount of resources and a range of deviations of the decision variables--were well suited to the Dynamic Programing solution. Through use of the linear programing methodology, CAA formulated a force for CONAF IV that minimized the number of Blue troops on the battlefield with equal or greater Red tank and Red people kills. Conceptually this is only slightly different from the problem solved here. However, the dual constraint lead to technical problems in the solution procedure that have not been solved at this point in the USMA-CAA research.

c. Aviation companies were not used as a decision variable. This action should be undertaken to achieve a solution completely comparable with the CAA 1975 CONAF study. This aspect of research is, however, minor when compared with the two difficulties stated above.

3. Cost and Effectiveness Factors

a. Cost Factors. - The costing of the forces is probably the weakest part of both this study and the 1975 CAA CONAF study.* The question, to some extent, seems absurdly simple: "How much does it cost to add one tank battalion to the Base Case"? Yet, as indicated by an elementary approach to costing, the technique of answering this question can be laborious. A systematic approach to these cost determinations appears a must for future studies.

b. Effectiveness Factors. - The derived effectiveness factors are only slightly more satisfying than the cost factors. Again, a systematic analysis of the CEM runs is needed to precisely evaluate effectiveness factors. The use of multiple regression analysis is suggested to delineate the causative factors in this complex model. An input-output relationship of this type would equate to a mini-CEM and yield at least "ball park" figures that may prove satisfactory for many studies that CAA may undertake. The saving in computer time from not having to run the full CEM model might more than offset the analytical effort required to determine the effectiveness factors.

4. Computer Programs

The computer program at Appendix F is completely satisfactory for the present limited objectives although further exercise of the program by CAA analysts may dictate change.

*Concepts Analysis Agency, "Conceptual Design for the Army in the Field, Phase IV (CONAF IV) (U)," July 1975.

CHAPTER VI
OBSERVATIONS

1. Observations

a. The following general observations were made during the conduct of the joint USMA-CAA study of a Dynamic Programing approach to Army force planning.

(1) The Dynamic Programing method, in general, is suited for force planning and offers significant advantages over trial and error or linear programing solutions.

(2) There is a class of possible problems involving multiple objectives and/or multiple constraints that are not susceptible to Dynamic Programing solutions at the present state of USMA-CAA development.

(3) Future extensions suggested for CAA considerations are: Expansion of criteria and decision variables. Improvement in cost and effectiveness factors, and modification of computer programs.

(4) The CAA cost and effectiveness data base is not entirely satisfactory for either linear programing or Dynamic Programing solutions. A systematic analysis of the CEM runs is needed to precisely evaluate cost and effectiveness factors. With these improvements CAA could well use Dynamic Programing routinely in future CONAF studies.

b. Specific observations were made in the application of the Dynamic Programing approach to the "Solution of a Sample Problem," and in the determination of "A European Oriented Optimal Force."

(1) "Solution of a Sample Problem".

a. Dynamic Programing always tells the decision maker more than just the answer to one specific question and provides a veritable storehouse of information for post hoc analysis or answers to "what if" questions.

b. Almost unlimited variations of single constraints could be considered when Dynamic Programing is used to determine, for example, optimal force mix.

c. If the analyst is astute in his choice of decision variables, the answers will be in terms completely familiar to the decision maker. Translation problems are not encountered and all constraints are rigorously met.

(2) <u>"A European Oriented Optimal Force"</u>.

a. The two constraints of the European force problem--a fixed dollar amount of resources and a range of deviations of the decision variables--were well suited to the Dynamic Programing solution.

b. In this exercise tank kills are maximized by increasing the number of tank battalions in the authorized force.

c. The alternative optimal force is relatively insensitive to the type of curve fit used. Although an exponential curve has mathematically satisfying properties indications are that an analyst should not be reluctant to turn to the expedient linear fit.

d. Only for the hyperbolic curve fit case, where the artillery battalions dip to 148, is any constraint "tight." Part of this may be attributed to "round-off" of costs in the Dynamic Programing solution. There is a mild presumption that loosening the constraint, perhaps to ± 30 percent, or even ± 50 percent, would not result in a radically different force.

APPENDIX A

STUDY CONTRIBUTORS

NOT USED

A DYNAMIC PROGRAMING APPROACH TO ARMY FORCE PLANNING

APPENDIX A
STUDY CONTRIBUTORS

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NOT USED

APPENDIX B

GLOSSARY

NOT USED

APPENDIX B
GLOSSARY

APC	armored personnel carrier
CAA	Concepts Analysis Agency
CEM	CONAF Evaluation Model
CONAF	conceptual design for the Army in the field
FEBA	forward edge of the battle area
$f(\underline{s})$	value of the return function at point $\underline{s}$
FY	fiscal year
$\underline{s}$	collective or vector representation of values
T	transformation, e.g., T_1, one of six alternative choices
USMA	United States Military Academy
X_1	number of infantry brigades
X_2	number of armor brigades
X_3	number of cavalry regiments
$	dollars

NOT USED

APPENDIX C

REFERENCES

NOT USED

APPENDIX C
REFERENCES

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NOT USED

APPENDIX D

COST CALCULATIONS

NOT USED

A DYNAMIC PROGRAMING APPROACH TO ARMY FORCE PLANNING

APPENDIX D
COST CALCULATIONS

1. Purpose

The purpose of this Appendix is to describe the costing rules and calculations, and to illustrate the use of these rules and calculations with a sample problem.

2. Fixed Costs

a. Battalion Equivalent Determination

The FY 87 Base Case consisted of:

Infantry battalions	-	89
Mechanized battalions	-	88
Tank battalions	-	85
Artillery battalions	-	188
Armored cavalry squadrons	-	15
Air cavalry squadrons	-	6
Subtotal	-	471

There were also:

Attack helicopter companies	-	11
Assault helicopter companies	-	33
Air cavalry troops	-	15
Total	-	59

The aggregate 59 aviation companies and troops are approximately equivalent to 20 battalions. The conclusion is that the Base Case force is:

471 battalions + 20 battalion equivalent = 491 total battalions.

b. Linear Support Costs

Personnel at Concepts Analysis Agency, using the Force Analysis Simulation of Theater Administrative and Logistics Support (FASTALS) Model, developed a troop list for support above division level, for the FY 1987 Base Case force. The CAA costed this Base Case and determined that the total cost (nonrecurring cost plus one year of recurring cost) for this support force was \$40.132 billion. For want of a more rational or precise approach, the assumption is made that these costs are linearly apportioned to each combat battalion, i.e.,

$$\frac{\$40.132 \text{ billion}}{491 \text{ battalions}} = \$81.73 \text{ million/battalion for support.}$$

In other words, to add one combat battalion to the Base Case increases the Base Case cost by $81.73 million. Conversely, this amount is saved by deletion of one combat battalion.

3. Variable Costs

a. Methodology

The variable costs are determined by detailed examination of the actual unit structure. From this determination rules are formulated that are consistent with organizational and support requirements. The assumption is made that the rules for adding a battalion are the same as the rules for deleting a battalion. The rules for each type battalion follow.

b. Infantry Battalions

There are nine division size units that are infantry heavy. The range of consideration for infantry battalions is $\pm$ 20 percent. For one to nine battalions one battalion is added to or substracted from these divisional size units. For 10 to 18 battalions, the deletion or addition is made of a two battalion brigade structured as:

1 Brigade headquarters company
2 Infantry battalions
1 Direct support artillery battalion (105 Towed)
1 Combat engineer company (infantry division)
1 Signal company
1 Medical company
1 Direct support supply and transportation company
1 Direct support maintenance company (infantry division)

For battalions 19 and 20, assume two brigades of three battalions each.

c. Mechanized Battalions

The range of the mechanized battalion is also $\pm$ 20 battalions. There are 11 divsion size units that are susceptible to tailoring. The rules then are similar to those for the infantry battalions. For changes of one to eleven battalions, add or substract the cost of one mechanized battalion. For battalions 12 to 20, configure a mechanized brigade as:

1 Mechanized brigade headquarters
2 Mechanized battalions
1 Direct support artillery battalion (155 SP)
1 Armored engineer company
1 Signal company
1 Medical company
1 Direct support supply and transportation company
1 Direct support maintenance company (armored)

d. Tank Battalions

The range of the tank battalions is also ± 20. However, there are only five division size units that are sufficiently tank heavy to tailor. Therefore, the rule is (for battalions one to five) add or delete one battalion from each of these five divisional size units. For battalions six through ten, assume a brigade exactly the same as the mechanized brigade except an armor brigade headquarters and tank battalions are employed in lieu of mechanized battalions. For battalions 11 through 15, structure a two-tank battalion brigade. Changes 16 through 20 are achieved by reorganizing separate armor cavalry regiments, and adding or deleting one battalion to five of these units.

e. Armored Cavalry Squadrons

The range is only ± 3 and the change is achieved by single unit additions or deletions.

f. Artillery Battalions

The range here is large, ± 40 battalions. However, all changes are effected at Corps or Army level. Therefore, the costs are assumed proportional to the number of battalions changed.

4. Sample Calculations

The costing methodology is illustrated by costing the mechanized battalions. For battalions 1 to 11 the rule calls for the addition or deletion of one battalion. Thus, for each of these battalions, the cost is:

Cost (nonrecurring plus 1-year recurring cost) of the battalion	-	\$ 6.62 million
Pro rata support cost	-	81.73 million
Total	-	\$88.35 million

For battalions 12 through 20, add or subtract a mechanized brigade. The costs are:

Brigade headquarters	-	\$ 1.377 million
Combat engineer company	-	1.725
Signal company	-	1.784
Medical company	-	0.241
Supply and transportation company	-	0.535
Maintenance company	-	0.949
Direct support artillery		
Battalion unit cost	-	8.830
Pro rata support cost		81.730
Mechanized battalion		
Battalion unit cost	-	6.620
Pro rata support cost		81.730
Total	-	\$185.820 million

5. Cost Results

a. Infantry Battalions

(1) battalions	1 through 9	$ 83.80*
(2) battalions	10 through 18	192.48
(3) battalions	19 through 20	83.80

b. Mechanized Battalions

(1) battalions	1 through 11	88.35
(2) battalions	12 through 20	185.82

c. Tank Battalions

(1) battalions	1 through 5	119.58
(2) battalions	6 through 10	234.01
(3) battalions	11 through 20	119.58

d. Artillery Battalions - $83.82 million/battalion for all battalions.

e. Armored Cavalry Squadrons - $84.82 million/squadron for all squadrons.

*All costs in millions of dollars

APPENDIX E

EFFECTIVENESS FACTORS

NOT USED

A DYNAMIC PROGRAMING APPROACH TO ARMY FORCE PLANNING

APPENDIX E
EFFECTIVENESS FACTORS

1. Purpose

The purpose of this Appendix is to describe the development and calculation of the effectiveness factors that were used in the Dynamic Programing method.

2. Data Base

a. Raw Data

The CONAF Evaluation Model (CEM) is an exceedingly complicated and sophisticated model that yields a surfeit of data--much of the data (as is typical of most analysis) is in not quite the format that is required for the Dynamic Programing solution. The analyst has a choice, for example, of:

(1) Red tanks killed on a day-by-day basis.
(2) Red tanks killed on a theater cycle (4-day) basis.
(3) Average daily Red tanks killed on a 30, 60, or 90-day basis.
(4) Average daily Red tanks killed on a "complete" war (96-day) basis.
(5) Blue tanks authorized on a day-by-day basis.
(6) Blue tanks authorized on a theater cycle basis.
(7) Average Blue tanks authorized on a 30, 60, 90 or 96-day basis.
(8) Blue tanks on-hand on a day-by-day basis.
(9) Blue tanks on-hand on a theater cycle basis.
(10) Average Blue tanks on hand on a 30, 60, 90 or 96-day basis.

A similar array exists for the other five weapon slices (armored personnel carriers (APC), helicopters, mortars, antitank weapons, and artillery tubes).

b. Translated Data

(1) Which then are the most appropriate data for this study? The problem statement required the optimal force for a 96-day war. A force that is optimal for a 30-day war may be far different than the force that is optimal for a 96-day war. Additionally, the force that is optimal for Day 1 of a 96-day war may be quite different than the force that is optimal on Day 96 of the war. However, the decision maker cannot be furnished 96 answers. The decision maker wants to

know what is recommended for the organization of the Army in FY 87. Therefore, average daily figures for the effectiveness factors (Red tanks killed, Red personnel killed, etc.) are chosen.

(2) The rational for the choice of average authorized strength or average on-hand strength parallels the discussion of effectiveness factors but is more subtle. Clearly, only a tank on the battlefield can kill a tank. "Paper" tanks do not kill "real" tanks. With this limited viewpoint one would opt for on-hand strengths. An analyst must be acutely aware of the decision the study is meant to influence. The CONAF studies are long range planning tools that furnish guidance for personnel and hardware procurement, needed legislation, and resource requirements. In shorthand, CONAF tells the decision maker what force the decision maker should buy in FY 87. Therefore, the 96-day average authorized strength was elected for the effectiveness calculations.

(3) The procedure is not yet complete. The decision variables are tank battalions, infantry battalions, and the like. The raw data is by weapon slice. The curve fitting procedure is discussed in detail in paragraph 3. Suffice it to say at this point that the effectiveness of a tank battalion is calculated by determining the number of tanks, APC, and antitank weapons in the battalion and then summing the respective average daily kills by each of these three types of weapons.

c. Theoretical Basis

An intuitive approach to the theoretical problem of determining the effectiveness of a tank battalion against enemy tanks would be to change the Base Case force by one tank battalion while holding everything else constant, then run the CEM and make the appropriate statistical comparison. However, this naive approach neglects the "black box" mystery of the CEM and the subtlety and sophistication of the model. The analyst has control over the input to the CEM and to that extent his control over "constancy." The analyst has little control over the inner mechanisms of the model. Hypothesize that one tank battalion of interest is added. Suppose that this additional battalion causes a delay in arrival in theater of an ammunition handling company. Thus, the addition of a tank battalion, may result not in greater capability but in an across-the-board degradation. Also cases could be pictured where the addition or deletion of one battalion would cause force ratio thresholds to be exceeded, and the entire tempo of combat might change with wildly unpredictable results. A battalion-by-battalion variation would require over 200 CEM runs for solution of this problem. The variation would be prohibitively expensive. Nine CEM runs existed that were generated for CONAF IV. Faced with the theoretical and practical difficulties of ideal data collection, the acceptance of extant data as typical cases and the fitting of curves to the data was forced. The curve fitting is described in the following paragraphs.

3. Curve Fitting Procedures

a. The Ideal Curve. - Ideally a curve such as Figure 7, that shows a decreasing return to scale and is asymptotic to some value (the maximum average number of Red tanks that could be killed) would be sought. Actually used and described are curve fits for three cases: a straight line, a hyperbola, and an exponential curve. Each of these is fit by the method of least squares.

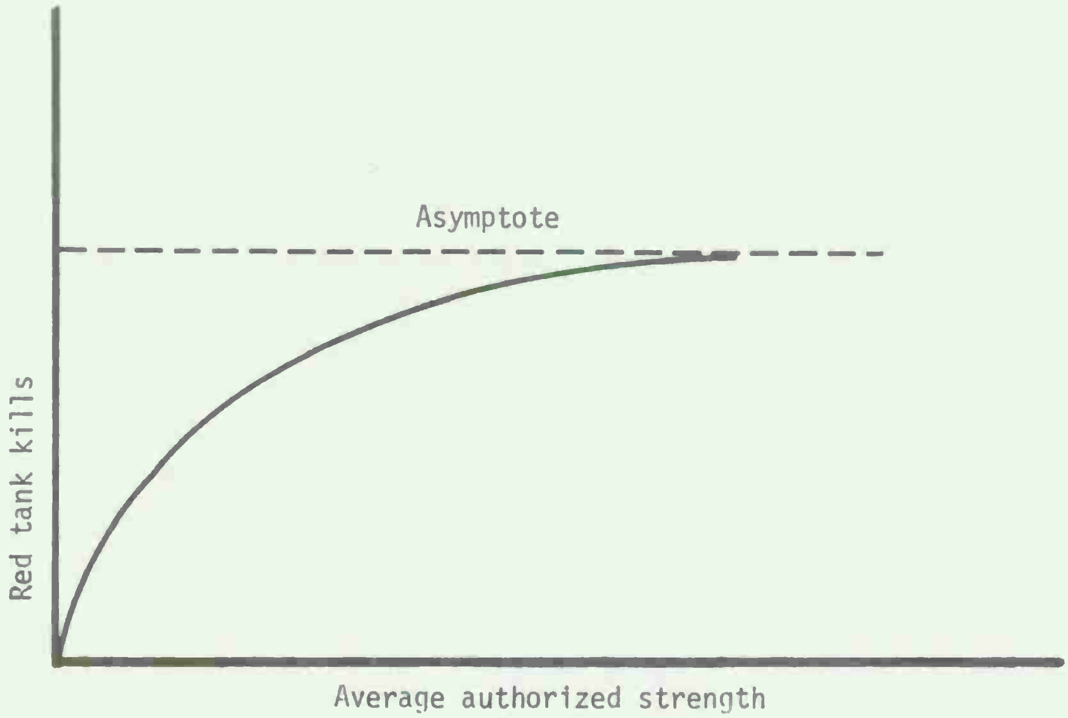

FIGURE 7, Theoretical Effectiveness Curve

b. The Straight Line. - The data was first fit to a straight line such as shown in Figure 8. The straight line obviously violates the criteria of decreasing return to scale and being asymptotic. In addition, it is quite likely the Y-axis will be intercepted at some positive value of "b" that implies the getting something for nothing, or at some negative value of "b" that implies that the first few tank battalions are totally ineffective. However, a defense of the straight line approximation is illustrated by Figure 9. The optimal force is constrained to deviate no more than ± 20 percent from Base Case force. Forces at or

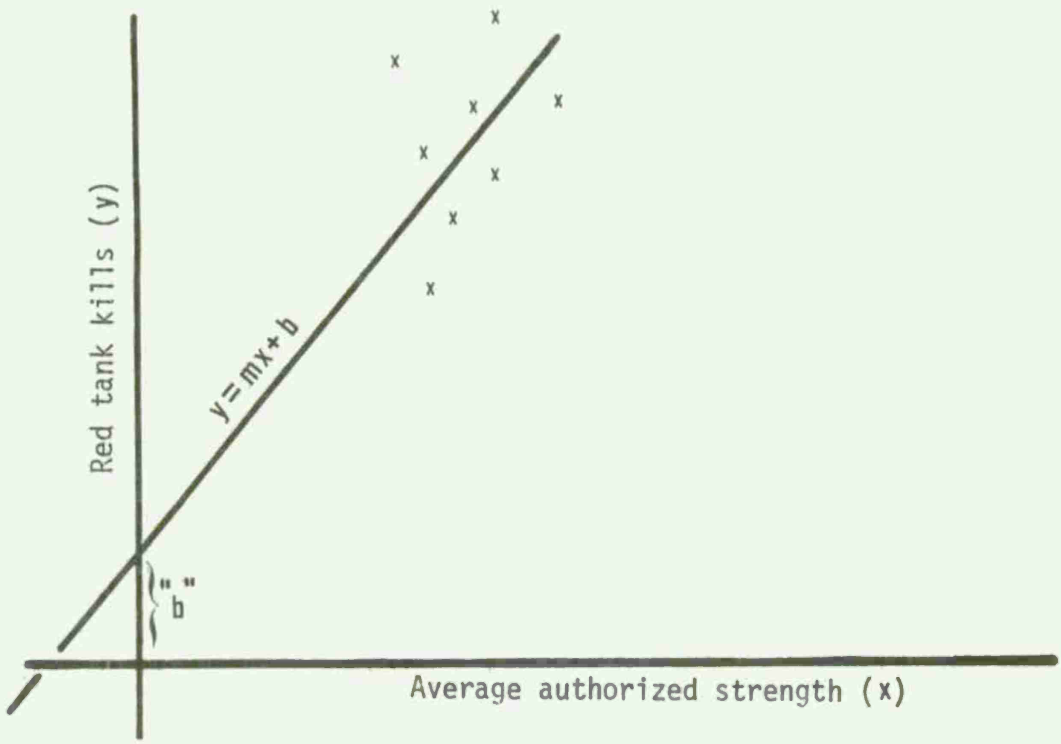

FIGURE 8, Straight Line Fit

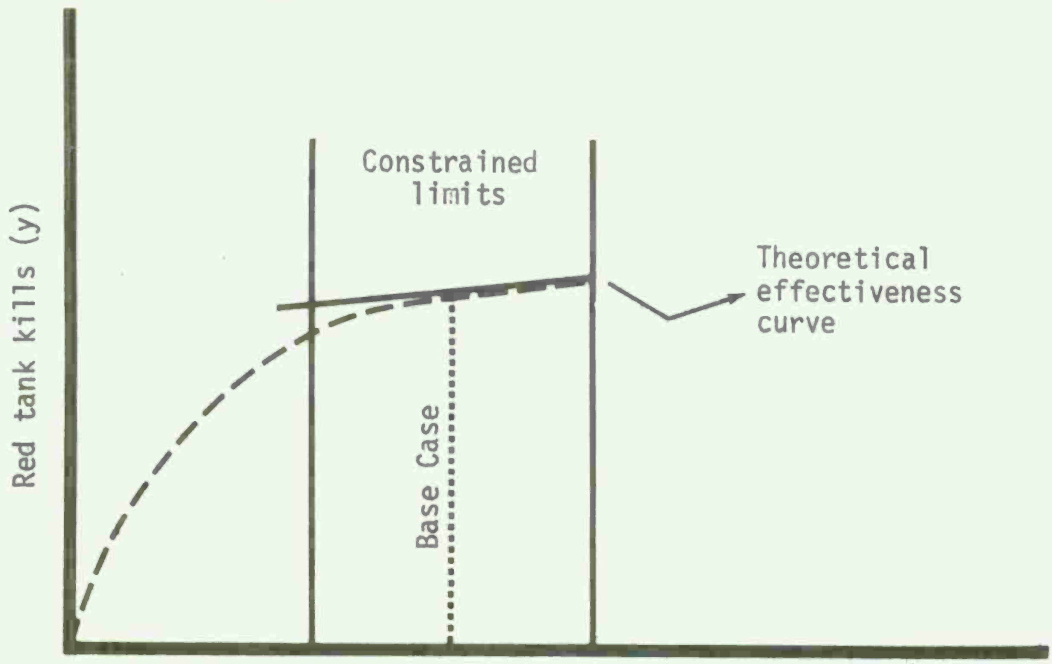

FIGURE 9, Linear Approximation

near the origin are constrained from entry into the solution. The only interest is in representing the effectiveness curve by a straight line over a very small region and with this limitation the straight line approximation may be quite sound. The results of the straight line fits are:

(1) Average daily Red tanks (Y) killed by Blue tanks (X)

$$Y = 0.031X - 6.53$$

(2) Average daily Red tanks (Y) killed by Blue APC (X)

$$Y = 0.0024X + 3.99$$

(3) Average daily Red tanks (Y) killed by Blue Antitank weapons (X)

$$Y = 0.012X + 127.8$$

(4) Average daily Red tanks (Y) killed by Blue artillery (X)

$$Y = 0.0011X - 0.534$$

(5) Average daily Red tanks (Y) killed by Blue helicopters (X)

$$Y = 0.0456X - 16.75$$

c. <u>A Hyperbolic Effectiveness Curve</u>

Figure 10 depicts a typical hyperbolic curve fit. The hyperbolic is a decreasing return to scale curve and is assymptotic. The curve fit determines values of "a" and "b". The results of this fit are:

(1) Average daily Red tanks (Y) killed by Blue tanks (X)

$$Y = 299.8 - 734604/X$$

(2) Average daily Red tanks (Y) killed by Blue APC (X)

$$Y = 42.3 - 150249/X$$

(3) Average daily Red tanks (Y) killed by Blue antitank weapons (X)

$$Y = 342.6 - 935157/X$$

(4) Average daily Red tanks (Y) killed by Blue artillery (X)

$$Y = 3.94 - 4580/X$$

(5) Average daily Red tanks (Y) killed by Blue helicopters (X)

$$Y = 37.44 - 15445/X$$

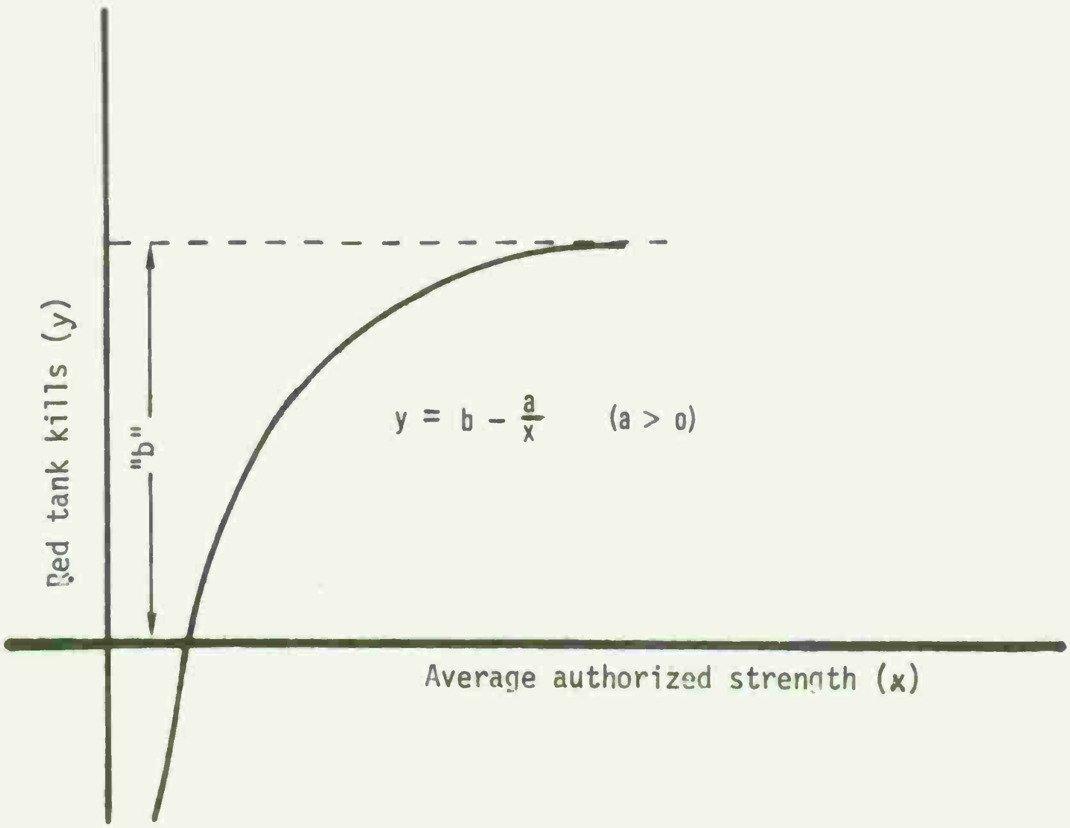

FIGURE 10, A Hyperbolic Effectiveness Curve

d. <u>An Exponential Effectiveness Curve</u>

The process is repeated with an exponential effectiveness curve (Figure 11). The results are:

(1) Average daily Red tanks (Y) killed by Blue tanks (X)

$$Y = 421.4 \exp(-5121.561/X)$$

(2) Average daily Red tanks (Y) killed by Blue APC (X)

$$Y = 52.98 \exp(-65.67.04/X)$$

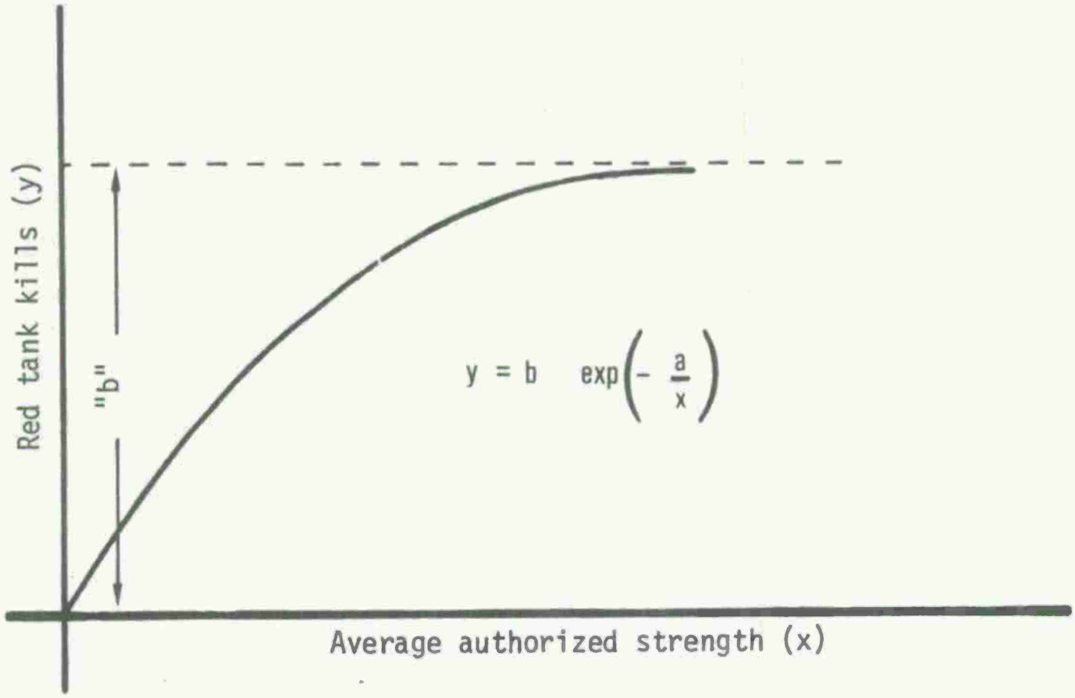

FIGURE 11, An Exponential Effectiveness Curve

(3) Average daily Red tanks (Y) killed by Blue Antitank weapons (X)

$$Y = 369.41 \exp(-3979.57/X)$$

(4) Average daily Red tanks (Y) killed by Blue artillery (X)

$$Y = 7.024 \exp(-3045.91/X)$$

(5) Average daily Red tanks (Y) killed by Blue helicopters (X)

$$Y = 143.06 \exp(-1584.61/X)$$

NOT USED

APPENDIX F

COMPUTER PROGRAMS

NOT USED

```
C DYNMC      DYNMC  = DYNAMIC PROGRAMMING METHOD
C   .   .   .   .   .   .   .   .   .   .   .   .   .   .
C
C       N    =  AN INTEGER VARIABLE SPECIFYING THE NUMBER OF ACTIVITIES.
C               N MUST NOT EXCEED 30.
C
C       NINT =  AN INTEGER VARIABLE SPECIFYING THE NUMBER OF INTERVALS
C               INTO WHICH THE RANGE BETWEEN 0 AND XMAX IS TO BE DIVIDED
C               TO OBTAIN  DELTA.  NINT MUST NOT EXCEED 1000.
C
C       XMAX =  THE TOTAL QUANTITY OF RESOURCE TO BE DISTRIBUTED BETWEEN
C               N ACTIVITIES.
C
C   .   .   .   .   .   .   .   .   .   .   .   .   .   .
C
C       REQUIREMENTS-
C               DYNMC REQUIRES THE PRESENCE OF THE FUNCTION SUBROUTINE
C               G(J,B).  G(J,B) PROVIDES THE ACTIVITY VALUE WHERE J
C               SPECIFIES THE ACTIVITY NUMBER AND B SPECIFIES THE ALLOCA-
C               TION QUANTITY TO THAT ACTIVITY.
C
C       STORAGE-
C               DYNMC REQUIRES 40K
C
C   .   .   .   .   .   .   .   .   .   .   .   .   .   .
C
C       X(I,J)= AN ELEMENT OF AN ARRAY CONTAINING THE ALLOCATION TO THE
C               ITH ACTIVITY WHICH MAXIMIZES THE SUM OF THE (ITH ACTIVITY
C               AND THE MAXIMUM OF THE SUM OF THE (I-1 ACTIVITIES))
C               GIVEN (J-1)*DELTA AS THE TOTAL ALLOCATION.
C
C       FO   =  AN ARRAY CONTAINING THE MAXIMUM VALUE OF THE SUM OF I-1
C               ACTIVITIES FOR THE SET OF TOTAL ALLOCATIONS, 0, DELTA,
C               2*DELTA, ..., NINT*DELTA=XMAX.
C
C       FI   =  AN ARRAY CONTAINING THE MAXIMUM VALUE OF THE SUM OF I
C               ACTIVITIES FOR THE SET OF TOTAL ALLOCATIONS, 0, DELTA,
C               2*DELTA, ..., NINT*DELTA=XMAX.
C
C       GP   =  AN ARRAY CONTAINING THE VALUE OF THE ITH ACTIVITY FOR
C               EACH ALLOCATION, 0, DELTA, 2*DELTA, ..., NINT*DELTA=XMAX.
C
C       SOL  =  OPTIMAL SOLUTION VALUE.
C
C       REMAIN= REMAINING RESOURCE.
C
C   .   .   .   .   .   .   .   .   .   .   .   .   .   .
C
C       D. R. HALE , 8/15/75
C
C   .   .   .   .   .   .   .   .   .   .   .   .   .   .
        SUBROUTINE DYNMC (N,NINT,XMAX)
        DIMENSION X(30,1001), FO(1001), FI(1001), GP(1001)
        INTEGER X,GAMMA
        IF (N.LE.0.OR.N.GT.30) GO TO 260
        IF (NINT.LE.0.OR.NINT.GT.1000) GO TO 260
        NINTP1=NINT+1
```

```
      XINT=FLOAT(NINT)
      DELTA=XMAX/XINT
      WRITE (6,270) N,XMAX,NINT,DELTA
C
C     FIRST ACTIVITY ALLOCATION
C
      DO 10 I=1,NINTP1
      IM1=I-1
      B=((6.*FLOAT(IM1))*XMAX/XINT)/16.
      F0(I)=G(1,B)
      X(1,I)=IM1
      IF (I.EQ.1) GO TO 10
      IF (F0(IM1).LE.F0(I)) GO TO 10
      F0(I)=F0(IM1)
      X(1,I)=X(1,IM1)
   10 CONTINUE
C**************************************************************************
C*    STAGE VARIATION LOOP.                                               *
C**************************************************************************
      DO 60 K=2,N
      KK=K
      DO 20 I=1,NINTP1
      IM1=I-1
      B=((6.*FLOAT(IM1))*XMAX/XINT)/16.
   20 GP(I)=G(KK,B)
      IF (K.EQ.N) GO TO 70
C**************************************************************************
C*    RESOURCE VARIATION LOOP                                             *
C**************************************************************************
      DO 40 J=1,NINTP1
      BETA=-1.E-20
C**************************************************************************
C*    ITERATION TO FIND X(K,J) MAXIMIZING RECURRENCE RELATION AT STAGE    *
C*    K.                                                                  *
C**************************************************************************
      DO 30 IX=1,J
      L=J-IX+1
      ALPHA=GP(IX)+F0(L)
      IF (ALPHA.LE.BETA) GO TO 30
      BETA=ALPHA
      GAMMA=IX-1
   30 CONTINUE
      F(J)=BETA
      X(K,J)=GAMMA
   40 CONTINUE
      DO 50 I=1,NINTP1
   50 F0(I)=F(I)
   60 CONTINUE
C
C     LAST ACTIVITY ALLOCATION
C
   70 BETA=-1.E-20
      J=NINTP1
      DO 80 IX=1,J
      L=J-IX+1
      ALPHA=GP(IX)+F0(L)
      IF (ALPHA.LE.BETA) GO TO 80
```

```
      BETA=ALPHA
      GAMMA=IX-1
   80 CONTINUE
      X(K,J)=GAMMA
C**************************************************************************
C*    BACK SOLUTION                                                       *
C**************************************************************************
      K=N
      WRITE (6,280)
      SOL=0.
   90 ALPHA=FLOAT(X(K,J))*XMAX/X(NT
      GO TO (100,150,180,230,240), K
  100 IF (ALPHA.GE.12166.) GO TO 110
      IF (ALPHA.GE.10881.) GO TO 120
      IF (ALPHA.GE.9566.) GO TO 130
      IF (ALPHA.GE.8396.) GO TO 140
      NU=(ALPHA-572.)/119.58
      GO TO 250
  110 NU=(ALPHA-572.)/119.58
      GO TO 250
  120 NU=(ALPHA-10298.)/234.0)
      GO TO 250
  130 NU=ALPHA/119.58
      GO TO 250
  140 NU=(ALPHA-9154.)/234.0)
      GO TO 250
  150 IF (ALPHA.GE.8835.) GO TO 160
      IF (ALPHA.GE.6802.) GO TO 170
      NU=(ALPHA-8799.)/202.63
      GO TO 250
  160 NU=(ALPHA-11313.)/202.63
      GO TO 250
  170 NU=ALPHA/88.35
      GO TO 250
  180 IF (ALPHA.GE.10137.) GO TO 190
      IF (ALPHA.GE.8296.) GO TO 200
      IF (ALPHA.GE.6704.) GO TO 210
      IF (ALPHA.GE.4971.) GO TO 220
      NU=(ALPHA-978.)/83.8
      GO TO 250
  190 NU=(ALPHA-978.)/83.8
      GO TO 250
  200 NU=(ALPHA-10650.)/192.48
      GO TO 250
  210 NU=ALPHA/83.8
      GO TO 250
  220 NU=(ALPHA-8694.)/192.48
      GO TO 250
  230 NU=ALPHA/83.32
      GO TO 250
  240 NU=ALPHA/84.82
      GO TO 250
  250 WRITE (6,290) K,NU
      SOL=SOL+G(K,ALPHA)
      J=J-X(K,J)
      K=K-1
      IF (K.NE.0) GO TO 90
```

```
      REMAIN=FLOAT(J-1)*XMAX/XINT
      WRITE (6,300) SOL,REMAIN
      RETURN
  260 WRITE (6,310)
      RETURN
C
C
C
  270 FORMAT (/'NUMBER OF ACTIVITIES =',I3/'THE RANGE BETWEEN 0 AND XMAX
     2           =',1PE12.4/'    IS DIVIDED INTO',I4,' INTERVALS.'/'DELTA
     3=',1PE12.4)
  280 FORMAT ('THE ALLOCATIONS ARE AS FOLLOWS')
  290 FORMAT ('X(',I2,') =',I4)
  300 FORMAT (/'THE OPTIMAL SOLUTION IS',1PE12.4/'THE REMAINING RESOURCE
     2           =',1PE12.4)
  310 FORMAT (1X,6HERROR))
      END
```

END ONSITE PRINTOUT ON DECEMBER 2, 1975 AT 14:32:23
UNCLASSIFIED*23HALE(1).DYNMC(3)

DYNAMIC PROGRAM DATE 120275 PAGE 9

```
C        *I RUNH * ENGINEERING/H/DYNMC,R    CARD REMOVED FROM THIS POINT
         CALL DYNMC (5,1000,45000.)
         STOP
C         ********* END CARD REMOVED FROM THIS POINT AT CAA *********
         FUNCTION G (J,R)
         XINCR=.1
         GO TO (10,80,130,200,220), J
   10    G=-1000.
         IF (R.GT.13129.+XINCR) RETURN
         IF (R.GE.12166.-XINCR) GO TO 20
         IF (R.GE.10881.-XINCR) GO TO 30
         IF (R.GE.9566.-XINCR) GO TO 40
         IF (R.GE.8396.-XINCR) GO TO 50
         IF (R.GE.7200.-XINCR) GO TO 60
         G=-1000.
         RETURN
   20    NU=(R-572.)/119.58
         GO TO 70
   30    NU=(R+10299.)/234.01
         GO TO 70
   40    NU=R/119.58
         GO TO 70
   50    NU=(R+9154.)/234.01
         GO TO 70
   60    NU=(R+572.)/119.58
         GO TO 70
   70    NT=54*NU
         NAP=18*NU
         NAT=8*NU
         PT=-6.53*(.031*NT)
         RAP=3.99*(.0024*NAP)
         RAT=127.8*(.012*NAT)
         G=RT+RAP+RAT
         RETURN
   80    G=-1000.
         IF (R.GT.10570.+XINCR) RETURN
         IF (R.GE.8835.-XINCR) GO TO 90
         IF (R.GE.6802.-XINCR) GO TO 100
         IF (R.GE.4979.-XINCR) GO TO 110
         G=-1000.
         RETURN
   90    NU=(R+11313.)/202.63
         GO TO 120
  100    NU=R/88.35
         GO TO 120
  110    NU=(R+8799.)/202.63
         GO TO 120
  120    NAP=63*NU
         NAT=62*NU
         RAP=.0024*NAP
         RAT=.012*NAT
         G=RAP+RAT
         RETURN
  130    G=-1000.
         IF (R.GT.10112.+XINCR) RETURN
         IF (R.GE.10137.-XINCR) GO TO 140
         IF (R.GE.8296.-XINCR) GO TO 150
```

```
      IF (B.GE.6709.-XINCR) GO TO 160
      IF (B.GE.4971.-XINCR) GO TO 170
      IF (B.GE.4809.-XINCR) GO TO 180
      G=-1000.
      RETURN
140   NU=(B-978.)/83.8
      GO TO 190
150   NU=(B+10650.)/192.48
      GO TO 190
160   NU=B/93.8
      GO TO 190
170   NU=(B+8699.)/192.48
      GO TO 190
180   NU=(B-978.)/83.8
      GO TO 190
190   NAT=54*NU
      RAT=.012*NAT
      G=RAT
      RETURN
200   G=-1000.
      IF (B.GT.14997.+XINCR) RETURN
      IF (B.GE.12331.-XINCR) GO TO 210
      G=-1000.
      RETURN
210   NU=B/83.32
      NAR=19*NU
      RAR=-.584*(.0011*NAR)
      G=RAR
      RETURN
220   G=-1000.
      IF (B.GT.1526.+XINCR) RETURN
      IF (B.GE.1017.-XINCR) GO TO 230
      G=-1000.
      RETURN
230   NU=B/84.82
      NT=27*NU
      NAP=75*NU
      NAT=22*NU
      NH=726*(8*NU)
      RT=.031*NT
      RAP=.0024*NAP
      RAT=.012*NAT
      RH=-(6.75*1.0*56*NH)
      G=RT+RAP+RAT+RH
      RETURN
```

END ONSITE PRINTOUT ON DECEMBER 2, 1975 AT 16:13:06
UNCLASS(FIED=3MALE(1).DTEST(1)(1)

```
      @I RUNM ,     ENGINEERING/M/DYNMC,R   CARD REMOVED FROM THIS POINT AT CAA
      CALL DYNMC (5,1000,45000.)
      STOP
      END    CARD REMOVED FROM THIS POINT AT CAA   ***************
      FUNCTION G (J,R)
      XINCR=.1
      GO TO (10,80,130,200,220), J
 10   G=-1000.
      IF (R.GT.13128.+XINCR) RETURN
      IF (R.GE.12166.-XINCR) GO TO 20
      IF (R.GE.10881.-XINCR) GO TO 30
      IF (R.GE.9566.-XINCR) GO TO 40
      IF (R.GE.8394.-XINCR) GO TO 50
      IF (R.GE.7200.-XINCR) GO TO 60
      G=-1000.
      RETURN
 20   NU=(R-572.)/119.58
      GO TO 70
 30   NU=(R+10298.)/234.01
      GO TO 70
 40   NU=R/119.58
      GO TO 70
 50   NU=(R+9154.)/234.01
      GO TO 70
 60   NU=(R+572.)/119.58
      GO TO 70
 70   NT=(5*NU)+150)
      NAP=(18*NU)+3197
      NAT=(8*NU)+1708
      RT=421.4*(EXP(-5121.56/NT))
      RAP=52.98*(EXP(-6567.04/NAP))
      RAT=369.41*(EXP(-3979.57/NAT))
      G=RT+RAP+RAT
      RETURN
 80   G=-1000.
      IF (R.GT.10570.+XINCR) RETURN
      IF (R.GE.8935.-XINCR) GO TO 90
      IF (R.GE.6802.-XINCR) GO TO 100
      IF (R.GE.4979.-XINCR) GO TO 110
      G=-1000.
      RETURN
 90   NU=(R-11313.)/202.63
      GO TO 120
 100  NU=R/88.35
      GO TO 120
 110  NU=(R+8799.)/202.63
      GO TO 120
 120  NAP=(63*NU)+4727
      NAT=(62*NU)+2348
      RAP=52.98*(EXP(-6567.04/NAP))
      RAT=369.41*(EXP(-3979.57/NAT))
      G=RAP+RAT
      RETURN
 130  G=-1000.
      IF (R.GT.10112.+XINCR) RETURN
      IF (R.GE.10137.-XINCR) GO TO 140
      IF (R.GE.8296.-XINCR) GO TO 150
```

DYNAMIC PROGRAM DATE 120275 PAGE 1

```
       IF (R.GE.6709.-XINCR) GO TO 160
       IF (R.GE.4971.-XINCR) GO TO 170
       IF (R.GE.4404.-XINCR) GO TO 180
       G=-1000.
       RETURN
  140  NU=(R-978.)/83.8
       GO TO 190
  150  NU=(R+1065.)/192.48
       GO TO 190
  160  NU=R/83.8
       GO TO 190
  170  NU=(R+8694.)/192.48
       GO TO 190
  180  NU=(R+978.)/83.8
       GO TO 190
  190  NAT=(54*NU)+7844
       RAT=342.6-(193515/NAT)
       G=RAT
       RETURN
  200  G=-1000.
       IF (R.GT.18997.+XINCR) RETURN
       IF (R.GE.17331.+XINCR) GO TO 210
       G=-1000.
       RETURN
  210  NU=R/83.32
       NAR=R*NU
       RAR=3.99-(15445/NAR)
       G=RAR
       RETURN
  220  G=-1000.
       IF (R.GT.1526.+XINCR) RETURN
       IF (R.GE.1017.-XINCR) GO TO 230
       G=-1000.
       RETURN
  230  NU=R/84.42
       NT=(27*NU)+1094
       NAP=(75*NU)+2072
       NAT=(22*NU)+1378
       NH=(8*NU)+726
       RT=299.8-(73484/NT)
       RAP=42.3-(150249/NAP)
       RAT=342.6-(193515/NAT)
       RH=37.44-(15445/NH)
       G=RT+RAP+RAT+RH
       RETURN
```

END ONSITE PRINTOUT ON DECEMBER 2, 1975 AT 18:34:26
UNCLASSIFIED*D3HALE(1).DTEST22(3)

DYNAMIC PROGRAM DATE 120275 PAGE 2

```
C      *I RUNH *    ENGINEERING/H/DYNMC.R  CARD REMOVED FROM THIS POINT
       CALL DYNMC (5,1000,45000.)
       STOP
C      END    CARD REMOVED FROM THIS POINT AT C&A ***********
       FUNCTION G (J,B)
       XINCR=.1
       GO TO (10,80,130,200,220), J
 10    G=-1000.
       IF (B.GT.13128.+XINCR) RETURN
       IF (B.GE.12166.-XINCR) GO TO 20
       IF (B.GE.10881.-XINCR) GO TO 30
       IF (B.GE.9566.-XINCR) GO TO 40
       IF (B.GE.8396.-XINCR) GO TO 50
       IF (B.GE.7200.-XINCR) GO TO 60
       G=-1000.
       RETURN
 20    NU=(B-572.)/119.59
       GO TO 70
 30    NU=(B+10294.)/234.01
       GO TO 70
 40    NU=B/119.58
       GO TO 70
 50    NU=(B+9154.)/234.01
       GO TO 70
 60    NU=(B+572.)/119.58
       GO TO 70
 70    NT=(54*NU)+1501
       NAP=(118*NU)+3107
       NAT=(8*NU)+1708
       RT=299.8-(173960/NT)
       RAP=42.3-(150249/NAP)
       RAT=342.6-(935157/NAT)
       G=RT+RAP+RAT
       RETURN
 80    G=-1000.
       IF (B.GT.10570.+XINCR) RETURN
       IF (B.GE.8835.-XINCR) GO TO 90
       IF (B.GE.6602.-XINCR) GO TO 100
       IF (B.GE.4979.-XINCR) GO TO 110
       G=-1000.
       RETURN
 90    NU=(B+11311.)/207.63
       GO TO 120
 100   NU=B/88.35
       GO TO 120
 110   NU=(B+9799.)/272.63
       GO TO 120
 120   NAP=(63*NU)+4727
       NAT=(62*NU)+2348
       RAP=42.3-(150249/NAP)
       RAT=342.6-(935157/NAT)
       G=RAP+RAT
       RETURN
 130   G=-1000.
       IF (B.GT.10112.+XINCR) RETURN
       IF (B.GE.10137.-XINCR) GO TO 140
       IF (B.GE.8296.-XINCR) GO TO 150
```

```
          IF (R.GE.6709.-XINCR) GO TO 160
          IF (B.GE.4971.-XINCR) GO TO 170
          IF (R.GE.4909.-XINCR) GO TO 180
          G=-1000.
          RETURN
    140   NU=(B-978.)/83.8
          GO TO 190
    150   NU=(R+10650.)/192.48
          GO TO 190
    160   NU=B/83.8
          GO TO 190
    170   NU=(B+8694.)/192.48
          GO TO 190
    180   NU=(R+978.)/83.8
          GO TO 190
    190   NAT=(54*NU)+7844
          RAT=369.91*(EXP(-3979.57/NAT))
          G=RAT
          RETURN
    200   G=-1000.
          IF (B.GT.18997.+XINCR) RETURN
          IF (R.GE.12331.-XINCR) GO TO 210
          G=-1000.
          RETURN
    210   NU=B/83.37
          NAR=18*NU
          RAR=7.024*(EXP(-3045.91/NAR))
          G=RAR
          RETURN
    220   G=-1000.
          IF (R.GT.1526.+XINCR) RETURN
          IF (R.GE.1017.-XINCR) GO TO 230
          G=-1000.
          RETURN
    230   NU=B/84.82
          NT=(27*NU)+1096
          NAP=(75*NU)+2072
          NAT=(22*NU)+1378
          NM=18*NU+726
          RT=421.4*(EXP(-5121.56/NT))
          RAP=52.98*(EXP(-6567.04/NAP))
          RAT=369.91*(EXP(-3979.57/NAT))
          RM=143.06*(EXP(-1584.61/NM))
          G=RT+RAP+RAT+RM
          RETURN

END ONSITE PRINTOUT ON DECEMBER 2, 1975 AT 16:35:04
UNCLASSIFIED*O3HALE(1).DTEST33(3)
```